Cambridge Assessment International Education

Cambridge Pre-U

MATHEMATICS **9794/01**

Paper 1 Pure Mathematics 1 **May/June 2022**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document consists of **13** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Question	Answer	Marks	Guidance
1(a)	$u_2 = 4$	**B1**	Correct u_2
	$u_3 = -2$, $u_4 = 1$	**B1ft**	Correct u_3 and u_4, FT their u_2
1(b)	$50 \div 3 = 16$ rem2	**M1**	Attempt to identify that 16 complete periods are needed (e.g. just $^{50}/_{3}$) Could be implied by method
	$\sum_{n=1}^{50} u_n = 16(1+4+(-2))+1+4$	**M1**	Attempt method to sum the first 50 terms – 16 complete periods plus two more of their terms
	$= 53$	**A1**	Obtain 53

Question	Answer	Marks	Guidance
2	$u^2 - 6u + 8 = 0$, with $u = x^{\frac{1}{4}}$	**M1***	Attempt to write equation as quadratic soi If $x^2 - 6x + 8 = 0$ then need $x = x^{\frac{1}{4}}$ soi
	$(u - 2)(u - 4) = 0$	**M1dep**	Attempt to solve quadratic
	$u = 2$, $u = 4$	**A1**	Obtain both correct roots of disguised quadratic
	$x = 2^4$, $x = 4^4$	**M1**	Attempt x, by raising at least one of their roots to the power 4
	$x = 16, 256$	**A1**	Obtain $x = 16$ and 256

Question	Answer	Marks	Guidance
3(a)	$x = 4\cos\left(\frac{1}{3}\pi\right) = 2$	**B1**	Correct Re component B0 if ± 2
	$y = -4\sin\left(\frac{1}{3}\pi\right) = -2\sqrt{3}$	**B1**	Correct Im component B0 if $\pm 2\sqrt{3}$ No need to write as $2 - 2\sqrt{3}\,\mathrm{i}$
3(b)	$\mathrm{i}z = 2\sqrt{3} + 2\mathrm{i}$	**B1ft**	Correct iz, soi, for their z OR $\dfrac{-8\mathrm{i}}{z}$
	$\dfrac{8\left(2\sqrt{3} - 2\mathrm{i}\right)}{\left(2\sqrt{3} + 2\mathrm{i}\right)\left(2\sqrt{3} - 2\mathrm{i}\right)}$	**M1**	Attempt to rationalise their denominator
	$\dfrac{16\sqrt{3} - 16\mathrm{i}}{12 + 4}$	**A1ft**	Correct denominator for their fraction (allow 12 + 4 for 16)
	$\sqrt{3} - \mathrm{i}$	**A1**	Correct final answer If using mod/arg form: **B1** – obtain iz as 4(cos $^{\pi}/_{6}$ + isin $^{\pi}/_{6}$) **M1** – attempt 8(cos 0 + isin 0) divided by their iz (must divide moduli and subtract arguments) **A1** – obtain 2(cos (- $^{\pi}/_{6}$) + isin (- $^{\pi}/_{6}$)) oe **A1** – obtain $\sqrt{3} - \mathrm{i}$

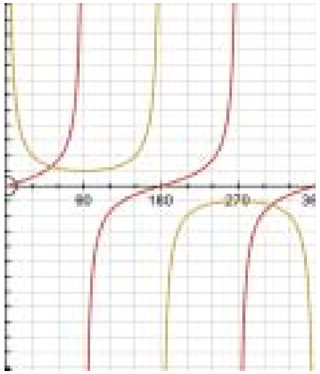
Question	Answer	Marks	Guidance
4(a)	$\frac{2}{5}\pi$	**B1**	State correct angle – allow any exact equiv
4(b)	$ABCD = \frac{1}{2}(r+2)^2 \times \frac{2}{5}\pi - \frac{1}{2}r^2 \times \frac{2}{5}\pi$	**M1**	Attempt area of sector at least once, using correct expression, with numerical θ in radians
	$\frac{1}{2}(r+2)^2 \times \frac{2}{5}\pi - \frac{1}{2}r^2 \times \frac{2}{5}\pi = 8\pi$	**A1**	Obtain correct, unsimplified, equation
	$\frac{1}{5}\pi\left(r^2 + 4r + 4 - r^2\right) = 8\pi$ $4r + 4 = 40$	**M1**	Expand and attempt to solve their attempt at the difference of two sectors
	$r = 9$	**A1**	Obtain $r = 9$

Question	Answer	Marks	Guidance
5(a)	(r =) (**i** – 2**j** + **k**) + μ (7**i** + 14**j** – 7**k**)	**M1***	Attempt an equation for line AB of form (position vector) + μ (attempt at direction vector) Condone λ being used again
		A1	Obtain a correct equation – condone no r =
	$3 - \lambda = 1 + 7\mu$ $10 + 2\lambda = -2 + 14\mu$ $3 + 3\lambda = 1 - 7\mu$	**M1dep**	Set up at least two simultaneous equations (could have original direction vector for AB, or a simplified one)
	$\lambda = -2$ or $\mu = \frac{4}{7}$	**M1**	Solve for either λ or μ
	C is (5, 6, –3)	**A1**	Obtain correct point of intersection www Condone position vector not coordinate

Question	Answer	Marks	Guidance
5(b)	$\lvert AC\rvert = \sqrt{96} = 4\sqrt{6}$, $\lvert CB\rvert = \sqrt{54} = 3\sqrt{6}$ OR AC = 4**i** + 8**j** – 4**k**, CB = 3**i** + 6**j** – 3**k**	**M1**	Attempt both lengths, or both vectors, using their C
	ratio $AC : CB = 4 : 3$	**A1**	Obtain correct ratio – allow $1:\frac{3}{4}$ or $\frac{4}{3}:1$

Question	Answer	Marks	Guidance
6(a)	ff(16) = f(11)	**M1**	Attempt ff(16)
	= –24	**A1**	Obtain –24
6(b)	$y = (x-14)^3 + 3$ $y - 3 = (x-14)^3$ $x - 14 = \sqrt[3]{y-3}$ $x = 14 + \sqrt[3]{y-3}$	**M1**	Attempt correct process to find inverse
	$\mathrm{f}^{-1}(x) = 14 + \sqrt[3]{x-3}$	**A1**	Obtain correct inverse function, in terms of x
	$x \in \mathbb{R}$	**B1**	Any correct statement indicating that $\mathrm{f}^{-1}(x)$ is defined for all real values of x
6(c)	$x_{n+1} = x_n - \dfrac{(x_n - 14)^3 + 3 - x_n}{3(x_n - 14)^2 - 1}$	**B1**	Correct Newton-Raphson formula soi
	18.14, 17.03, 16.51, 16.38, 16.37, 16.37…	**M1**	Attempt use of their N-R formula (at least 3 iterates)
	P is (16.37, 16.37)	**A1**	Obtain (16.37, 16.37), following at least 6 iterates Must be 4sf

Question	Answer	Marks	Guidance
6(d)	P is the point of intersection of $\mathrm{f}(x)$ and $\mathrm{f}^{-1}(x)$	**B1**	Any correct explanation relating to intersection of curves B0 for just $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$

Question	Answer	Marks	Guidance
7(a)		**M1**	Correct sketch of $y = \tan x$
		A1	Intercepts of (0, 0), (180, 0) and (360, 0) indicated
		M1	Correct sketch of $y = \operatorname{cosec} x$
		A1	(90°, 1) and (270°, –1) indicated **SC B1** not A0A0 if all labels correct but in radians
7(b)	$\dfrac{\sin x}{\cos x} = \dfrac{1}{\sin x}$	**B1**	Correct identity for cosec$x$ seen or implied
	$\sin^2 x = \cos x$ $1 - \cos^2 x = \cos x$	**M1**	Rearrange, and use $\sin^2 x = 1 - \cos^2 x$ (or equiv with $\cot^2 x$ or $\operatorname{cosec}^2 x$)
	$\cos^2 x + \cos x - 1 = 0$	**A1**	Obtain correct quadratic in cos$x$ (or any equiv correct equation in a single trig identity)
	$\cos x = \dfrac{-1 \pm \sqrt{5}}{2}$ $x = 51.8°$	**M1**	Solve quadratic for cos$x$, and attempt at least one value for x May not see negative value for cos$x$
	$x = 51.8°, 308.2°$	**A1**	Obtain both correct values (1dp or better), and no others in given range

Question	Answer	Marks	Guidance
8(a)(i)	$\left(\sqrt{a}-\sqrt{a+b}\right)\left(\sqrt{a}+\sqrt{a+b}\right)$ $=a+\sqrt{a}\sqrt{a+b}-\sqrt{a}\sqrt{a+b}-(a+b)$ $=-b$ **AG**	**B1**	Expand and obtain given answer Middle two terms may not be seen, but B0 if seen and incorrect
8(a)(ii)	$\dfrac{1}{\sqrt{a+b}}-\dfrac{1}{\sqrt{a}}=\dfrac{\sqrt{a}-\sqrt{a+b}}{\sqrt{a}\sqrt{a+b}}$	**B1**	Correctly combine as single fraction
	$=\dfrac{\left(\sqrt{a}-\sqrt{a+b}\right)\left(\sqrt{a}+\sqrt{a+b}\right)}{\sqrt{a}\sqrt{a+b}\left(\sqrt{a}+\sqrt{a+b}\right)}$ $=\dfrac{-b}{\sqrt{a}\sqrt{a}\sqrt{a+b}+\sqrt{a}\sqrt{a+b}\sqrt{a+b}}$	**M1**	Show intention to multiply top and bottom by $\sqrt{a}+\sqrt{a+b}$
	$=\dfrac{-b}{a\sqrt{a+b}+\sqrt{a}(a+b)}$ **AG**	**A1**	Simplify using part **(i)** to obtain given answer
8(b)	$\mathrm{f}'(x)=\lim\limits_{h\to 0}\dfrac{\frac{1}{\sqrt{x+h}}-\frac{1}{\sqrt{x}}}{h}$	**M1**	Attempt $\frac{1}{h}\left[\mathrm{f}(x+h)-\mathrm{f}(x)\right]$
	$\mathrm{f}'(x)=\lim\limits_{h\to 0}\dfrac{\frac{-h}{x\sqrt{x+h}+\sqrt{x}(x+h)}}{h}=\lim\limits_{h\to 0}\dfrac{-1}{x\sqrt{x+h}+\sqrt{x}(x+h)}$	**M1**	Attempt to simplify using expression from **(a)**, including cancelling h
	$\mathrm{f}'(x)=\dfrac{-1}{x\sqrt{x}+x\sqrt{x}}=\dfrac{-1}{2x\sqrt{x}}=-\dfrac{1}{2}x^{-\frac{3}{2}}$ **AG**	**A1**	Complete proof by considering lim $h\to 0$ to obtain given answer

Question	Answer	Marks	Guidance
9	$\int \frac{1}{y}\mathrm{d}y = \int \ln(x+2)\mathrm{d}x$	M1	Separate variables and attempt integration (of at least one side)
	$\int \ln(x+2)\mathrm{d}x = x\ln(x+2) - \int \frac{x}{x+2}\mathrm{d}x$	M1	Attempt integration by parts on ln(x + 2)
		A1	Obtain correct expression, in terms of x or u (if substitution already used)
	$= x\ln(x+2) - \int\left(1 - \frac{2}{x+2}\right)\mathrm{d}x$ $= x\ln(x+2) - x + 2\ln(x+2) + c$	M1	Attempt second integration Could use algebraic division or substitution (e.g. $u = x + 2$) to deal with the improper fraction
	$\ln y = x\ln(x+2) - x + 2\ln(x+2) + c$	A1	Correct equation, in terms of x, condone no + c Could be – (x + 2) not – x from using substitution
	ln27 = 3ln3 – 1 + c, hence c = 1	M1	Attempt c using (1, 27), following integration attempt
	$\ln y = (x+2)\ln(x+2) - x + 1$	A1	Correct equation – any equiv as long as ln27 and 3ln3 dealt with **Note:** Condone brackets and/or modulus throughout **OR** (if using substitution to replace ln term) **M1** – as main scheme **M1** – use e.g. u = ln(x + 2) and attempt to get integrand in terms of u **M1** – attempt integration by parts once (expect $u\mathrm{e}^u - \int \mathrm{e}^u \mathrm{d}u$) **A1** – integrate again to obtain $u\mathrm{e}^u - \mathrm{e}^u$ **A1** – correct equation in terms of x **M1, A1** – as main scheme

Question	Answer	Marks	Guidance
10	$an = -\frac{2}{3}$	**B1**	State $an = -\frac{2}{3}$
	$\frac{n(n-1)(n-2)a^3}{6} = -\frac{112}{81}$	**M1**	Attempt fourth term (possibly as part of a full expansion) Condone ax^3 not $(ax)^3$
		M1	Equate attempt at fourth term to $-\frac{112}{81}$
	$\frac{n(n-1)(n-2)}{6} \times \frac{-8}{27n^3} = -\frac{112}{81}$	**M1**	Eliminate one variable, from two equations both in a and n
	$n^2 - 3n + 2 = 28n^2$ $27n^2 + 3n - 2 = 0$ OR $a^2 + a - 6 = 0$ $(9n - 2)(3n + 1) = 0$	**A1**	Obtain correct quadratic (or possibly cubic) in either n or a – as far as like terms combined
	$n = \frac{2}{9}$ $n = -\frac{1}{3}$	**A1**	Obtain both correct values for n
	$a = -3$ $a = 2$	**A1**	Obtain both correct values for a

Question	Answer	Marks	Guidance
11(a)	$\frac{\mathrm{d}}{\mathrm{d}x}(\sec x \tan x) = \sec x \tan x . \tan x + \sec x . \sec^2 x$	**M1**	Attempt differentiation of sec$x$tan$x$ using product rule, or quotient rule if in terms of sin$x$ and cos$x$
		A1	Correct (unsimplified) derivative
	$= \sec x(\sec^2 x - 1) + \sec^3 x = 2\sec^3 x - \sec x$	**M1**	Use $\tan^2 x = \sec^2 x - 1$, or other relevant identity
	$\frac{\mathrm{d}}{\mathrm{d}x}(\ln(\sec x + \tan x)) = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$	**M1**	Attempt differentiation of ln(sec$x$ + tan$x$)
		A1	Correct (unsimplified) derivative
	$= \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x} = \sec x$	**M1**	Attempt to simplify using common factor **SR B1** not **M1A1M1** for those who just quote derivative
	$\frac{\mathrm{d}}{\mathrm{d}x}(\sec x \tan x + \ln(\sec x + \tan x)) = 2\sec^3 x - \sec x + \sec x$ $= 2\sec^3 x$ **AG**	**A1**	Combine terms to obtain given answer, with sufficient detail

Question	Answer	Marks	Guidance
11(b)	vol $= \pi\int\left(\frac{1}{1-x^2}\right)^2 \mathrm{d}x$	**B1**	State or imply correct volume (limits not required)
	$\pi\int\frac{1}{\left(1-\sin^2 u\right)^2}\cos u\mathrm{d}u$	**M1**	Attempt integrand in terms of u Condone no du
	$\pi\int\frac{1}{\left(\cos^2 u\right)^2}\cos u\mathrm{d}u = \pi\int\sec^3 u\mathrm{d}u$	**A1**	Obtain correct simplified integrand (condone no du) Allow $\frac{1}{\cos^3 u}$ for $\sec^3 u$ Condone no π
	vol $= \frac{1}{2}\pi\left(\sec u\tan u + \ln\left(\sec u + \tan u\right)\right)$	**A1**	Use result from **(a)** to obtain correct integral Condone no π
	$\frac{1}{2}\pi\left[\begin{array}{l}\left(\sec\frac{1}{6}\pi\tan\frac{1}{6}\pi + \ln\left(\sec\frac{1}{6}\pi + \tan\frac{1}{6}\pi\right)\right) - \\ \left(\sec 0\tan 0 + \ln\left(\sec 0 + \tan 0\right)\right)\end{array}\right]$	**M1**	Attempt use of limits of $\frac{1}{6}\pi$ and 0 (or use x limits in integral in terms of x) Allow BOD M1 if lower limit of 0 not explicitly used, but then A0 Condone no π
	$\frac{1}{2}\pi\left[\left(\frac{2}{\sqrt{3}}\times\frac{1}{\sqrt{3}} + \ln\left(\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right)\right) - \left(0 + \ln 1\right)\right]$ $= \frac{1}{2}\pi\left(\frac{2}{3} + \ln\sqrt{3}\right) = \frac{1}{2}\pi\left(\frac{2}{3} + \frac{1}{2}\ln 3\right) = \frac{1}{12}\pi\left(4 + 3\ln 3\right)$	**A1**	Obtain given answer, with sufficient working Minimum of $\frac{1}{2}\pi\left[\left(\frac{2}{3} + \ln\sqrt{3}\right) - 0\right]$ and then one extra step before given answer Must have π present when limits used